MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

EMT2046 – ENGINEERING MATHEMATICS IV

(BE, CE, EE, LE, MCE, NE, OPE, RE, TE)

1 JUNE 2017

2:30 PM – 4:30 PM

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This exam paper consists of **8 pages** (including cover page) with **four questions** and an **appendix** only.
2. Attempt **ALL questions**. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

(a) Consider the following linear programming (LP) problem,

$$\begin{aligned} \text{Maximize } z = \ & 2x_1 - x_2 + x_3 \\ \text{subject to: } \ & 3x_1 + x_2 + x_3 \le 60 \\ & x_1 - x_2 + 2x_3 \le 10 \\ & x_1 + x_2 - x_3 \le 20 \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

(i) Solve this LP problem using the simplex method. State the basic feasible solution for each iteration.

[12 marks]

(ii) Convert the primal problem above into its dual problem and then find the optimal solution and optimal value of the dual problem using the relationship between primal and dual problems.

[5 marks]

(b) Apply Kruskal's algorithm to reduce the network in **Figure Q1** below to a minimum spanning tree. Draw the final network with its surviving edges.

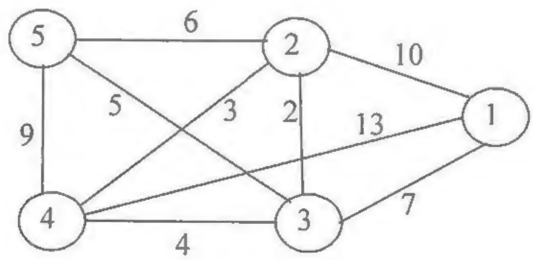

Figure Q1

[8 marks]

Continued...

Question 2

(a) Consider the following joint probability density function,

$$f_{XY}(x,y)=\begin{cases}\frac{6}{5}\left(x+y^2\right)dydx, & 0\le x\le 1, 0\le y\le 1\\ 0 & \text{otherwise.}\end{cases}$$

Find $E\left[g(x,y)\right]$ where $g(x,y)=40\left(x-y^2\right)$.

[5 marks]

(b) Suppose the one-step transition probability matrix of a Markov chain with state space $\{1, 2, 3, 4\}$ is given as follows,

$$\begin{bmatrix} 0.5 & 0 & 0.2 & 0.3 \\ 0.4 & 0 & 0.3 & 0.3 \\ 0.2 & 0 & 0.4 & 0.4 \\ 0.4 & 0 & 0.4 & 0.2 \end{bmatrix}$$

(i) Draw the state transition diagram.

[4 marks]

(ii) Determine all recurrent states.

[1 mark]

(iii) Find the period of each recurrent state.

[1 mark]

(iv) Find the probability to go from state 1 to state 3 in two transitions.

[2 marks]

(v) Suppose we place a particle in state 1 at time 0, what is the probability that the particle first reaches state 3 at time 2?

[2 marks]

(c) A woman gets her daily exercise by running (type I), swimming (type II) or biking (type III). She never runs or bikes two days in a row. If she runs one day, she is equally likely to swim or bike the next day. If she bikes, then the next day she is three times as likely to swim as to run. If she swims one day, then half the time she swims the next day and otherwise she is equally likely to run or bike. In the long run, what portion of her time is spent on each of the three activities? (Assume that the woman starts her exercise by running (type I).

[10 marks]

Continued...

Question 3

(a) Use the central-difference formula with step size h = 0.025 to estimate $f'(1.9)$ if $f(x) = e^x \sin(x)$.

[5 marks]

(b) Approximate the integral $\int_{0.2}^{2.6} e^{-x} dx$ by using the following methods,

(i) Composite trapezoidal rule with 6 intervals.

[6 marks]

(ii) Composite Simpson's rule with step size $h = 0.4$.

[6 marks]

(iii) Four-term Gaussian Quadrature.

[8 marks]

Continued...

Question 4

(a) The population of town A is as given below.

Year (x)	1951	1961	1971	1981	1991
Population (y) (in thousands)	46	66	81	93	101

By using all the data given and Newton's forward-difference formula, estimate the population of town A in the year 1955.

[11 marks]

(b) Apply Runge-Kutta method of order four with step size $h = 0.2$ for the differential equation $y' = y - x^2 + 1$ with initial condition $y(0) = 0.5$ to approximate $y(0.4)$.

[14 marks]

Continued...

APPENDIX: TABLE OF FORMULAS

1. The nth Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \ldots, x_k](x - x_0) \cdots (x - x_{k-1}).$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1)\ldots(x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0).$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n).$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

Continued...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

with the error term

$$\left|\frac{h^2}{6} f^{(3)}(c_x)\right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f(a) + f(b)\right).$$

The error term is $\left|\frac{h^3 f''(\xi)}{12}\right|$ for some ξ in (a, b) and $h = b - a$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{j=1}^{n-1} f(x_j)\right].$$

The error term is $\left|\frac{(b-a)h^2 f''(\xi)}{12}\right|$ for some ξ in (a, b) and $h = \frac{b-a}{n}$.

10. Simpson's rule

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right].$$

The error term is $\left|\frac{h^5}{90} f^{(iv)}(\xi)\right|$ for some ξ in (a, b) and $h = \frac{b-a}{2}$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j}) + 4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1}) + f(b)\right].$$

The error term is $\left|\frac{(b-a)h^4}{180} f^{(4)}(\xi)\right|$ for some ξ in (a, b), $h = \frac{b-a}{n}$, $x_j = a + jh$, $j = 0, 1, \ldots, n$.

Continued...

12. Four-term Gaussian Quadrature

$$\int_a^b f(x)\,dx = \frac{(b-a)}{2}\int_{-1}^{1} f\left(x = \frac{(b+a)+t(b-a)}{2}\right)dt$$

$$\int_{-1}^{1} f(x)\,dx \approx w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) + w_4 f(x_4)$$

Number of terms n	Nodes x_i	Weights w_i
4	-0.8611363116	0.3478548451
	-0.3399810436	0.6521451549
	0.3399810436	0.6521451549
	0.8611363116	0.3478548451

13. Newton-Raphson's method

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, \qquad n = 1,2,\ldots$$

14. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\frac{h^2}{2}Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}).

15. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

16. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1\right),$$
$$k_3 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2\right),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

End of Paper